except perhaps for a certain number of *finite discontinuities*. When V is of this kind, the condition (c) is necessary for consistency of the Schrödinger equation. To see this, let us consider the simplest case of a particle in one dimension, when the Schrödinger equation becomes

$$i\hbar \frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi \tag{2.45}$$

If ψ is a continuous function of x at all times in accordance with (c), then $\partial\psi/\partial t$ is also evidently a continuous function of x. Therefore, the right hand side of Eq. 2.45 must be continuous, and any departure from continuity in one of the two terms must be cancelled by an opposite behaviour in the other term. For example if V (and hence $V\psi$) has a finite discontinuity at some point a, $\partial^2\psi/\partial x^2$ also must have a finite discontinuity at a, i.e. $(\partial\psi/\partial x)$ must be continuous at a but its slope (which is $\partial^2\psi/\partial x^2$) to the right of a must be unequal to that on the left of a. (See Fig. 2.1). Conversely, if $(\partial\psi/\partial x)$ has this behaviour, V must necessarily have a finite discontinuity. If $(\partial\psi/\partial x)$ behaves any worse, e.g., if it has a finite discontinuity of its own [violating the condition (c)], then the curve for $\partial\psi/\partial x$ becomes 'vertical' at that point, and its slope, $\partial^2\psi/\partial x^2$ at that point is infinite. This would force V to have an *infinite* discontinuity. Conversely, we have:

(c′) *at any point where the potential V makes a sudden jump of infinite magnitude, $\partial\psi/\partial x$ has a finite discontinuity but ψ remains continuous.*

This statement is for the one-dimensional case. The generalization to three dimensions is straightforward, but we will not need it.

Any function which meets all the above requirements is *admissible as a wave function*. The requirements themselves are so natural as to be almost self-evident. But their consequences for the theory are of profound importance, as we shall see immediately.

C. STATIONARY STATES AND ENERGY SPECTRA

We have already noted that the state of a quantum mechanical system is specified by giving its wave function ψ. We are now in a position to see that in the case of time-independent systems (such as a particle moving in a *static* or time-independent potential V) there exist a special category of solutions of the wave equation, which describe *stationary* states. In these states, the position probability density $|\psi|^2$ at every point $\mathbf{x}$ in space remains independent of time; so also do the expectation values of all dynamical variables. Further when a particle is described by such a wave function its energy has a perfectly definite value. The energy spectrum (i.e. the set of energy values associated with the various stationary states) is, in general, at least partly discrete. We will now see (in the context of a simple example) how all these results follow in a very natural way from the Schrödinger equation and the admissibility conditions on wave functions. The existence of stationary states with discrete energies, which was *postulated* by Bohr in order to account for the nature of

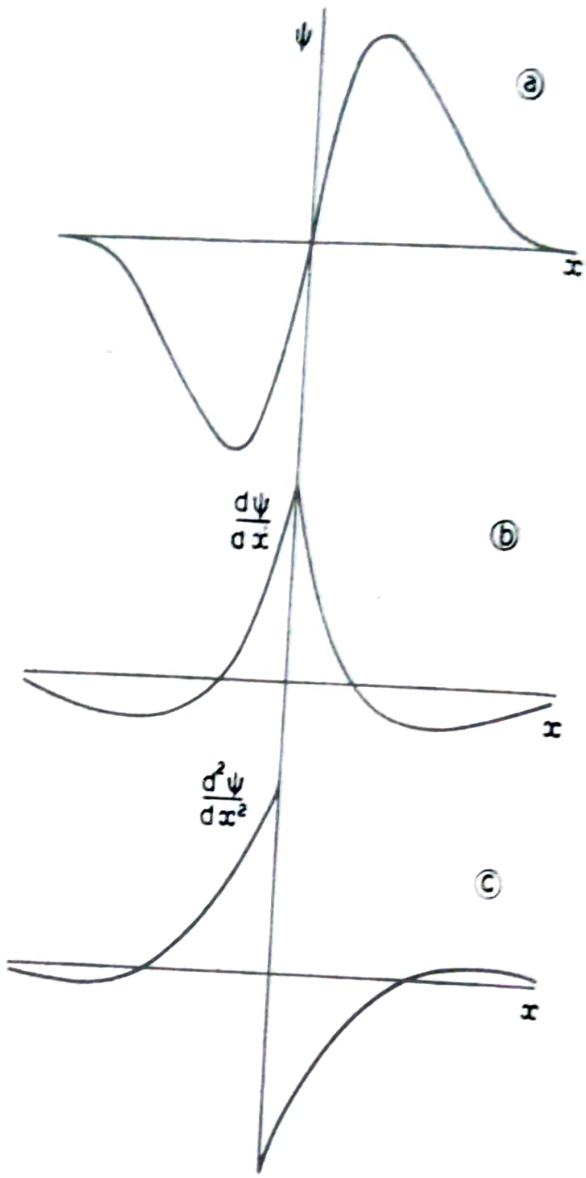

Fig. 2.1 Illustration of continuity properties (**a**) The wave function $\psi = e^{-|x|} \sin x$, continuous everywhere; (**b**) $d\psi/dx$: It has a 'cusp' at $x = 0$, but is still continuous; and (**c**) $d^2\psi/dx^2$: It has a finite discontinuity at $x = 0$.

atomic spectra, thus finds a rational explanation on the basis of quantum mechanics.

2.9 Stationary States; The Time-Independent Schrödinger Equation

Let us consider a particle moving in a *time-independent* potential $V(\mathbf{x})$. It is easy to verify that the Schrödinger equation in this case has solutions of the form

$$\psi(\mathbf{x}, t) = u(\mathbf{x}) f(t) \tag{2.46}$$

Substituting this assumed form in Eq. 2.17, and dividing throughout by $u(\mathbf{x}) f(t)$, we obtain

$$\frac{1}{f}\, i\hbar \frac{df}{dt} = -\frac{\hbar^2}{2m} \frac{\nabla^2 u}{u} + V(\mathbf{x}) \tag{2.47}$$

The right hand side of this equation is independent of t, and the left hand side is independent of $\mathbf{x}$. Their equality implies, therefore, that *both* sides must be independent of $\mathbf{x}$ and t, and hence must be equal to a constant, say E. Thus Eq. 2.47 separates into two equations

$$i\hbar \frac{df(t)}{dt} = E f(t) \tag{2.48}$$

and

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x})\right] u(\mathbf{x}) = E u(\mathbf{x}) \tag{2.49}$$

The former is readily solved, and shows that $f(t)$ is proportional to $\exp[-iEt/\hbar]$. The solution of Eq. 2.49 depends on the value assumed for E, and so we write it as $u_E(\mathbf{x})$. Thus Eq. 2.46 reduces to

$$\psi(\mathbf{x},t) = u_E(\mathbf{x})\, e^{-iEt/\hbar} \tag{2.50}$$

The value of E has to be real. For if it had an imaginary part ε, the wave function ψ would vanish for all $\mathbf{x}$ as $t \to \infty$ or $-\infty$ according as the sign of ε is $-$ or $+$, and this is of course not admissible. It follows then that

$$|\psi(\mathbf{x},t)|^2 = |u_E(\mathbf{x})|^2 \tag{2.51}$$

i.e. the probability density is independent of t. Similarly expectation values, as defined by Eq. 2.42, are also evidently time-independent. In other words, ψ of Eq. 2.50 describes a *stationary state* in which none of the particle characteristics changes with time.

Let us return now to Eq. 2.49 and the interpretation of E. This equation (called the *time-independent Schrödinger equation*) states that the action of the Hamiltonian operator of the particle (the quantity in square brackets) on the wave function $u_E(\mathbf{x})$ is simply to reproduce the same wave function multiplied by the *constant* E. This property is reminiscent of characteristic equations or eigenvalue equations in matrix theory, where a (column) vector u is called an eigenvector belonging to the eigenvalue λ of a matrix M if it satisfies the equation $Mu = \lambda u$ (with λ = a constant number). By analogy, we say that since $u_E(\mathbf{x})$ satisfies Eq. (2.49), it is an *eigen-function* belonging to the *eigenvalue* E of the (differential) operator $H = -(\hbar^2/2m)\nabla^2 + V$. Remember that this operator represents the energy of the particle as a function of the position and momentum. So $u_E(\mathbf{x})$ may be called an energy eigenfunction. Anticipating the general interpretation (Sec. 3.9) of eigenvalues and eigenfunctions, we now assert that *when the state of a particle is described by an energy eigenfunction, the energy of the particle has a definite value, given by the eigenvalue* E. The reader is invited to verify for himself that this interpretation is consistent with what he already knows about the free-particle case, by substituting the wave function (2.11) into Eqs. 2.48 and 2.49 with $V = 0$.

The following question still remains: If we assign an *arbitrary* real value to E in Eq. 2.49, does there exist a corresponding eigenfunction $u_E(\mathbf{x})$? The answer is in the negative. This is because the solution of Eq. 2.49 does not satisfy the admissibility conditions of Sec. 2.8, unless E is restricted to certain

specific values. Only these special values are to be considered as eigenvalues. The set of all such admissible values of E form what is called the eigenvalue spectrum of energy, or simply the *energy spectrum*. Following the terminology of old quantum theory, we shall frequently refer to the energy eigenvalues as *energy levels* of the system. We shall now determine the energy spectrum and energy eigenfunctions of a very simple system, in order to illustrate how the admissibility conditions on wave functions lead to restrictions on possible energy values.

2.10 A Particle in a Square Well Potential

The example we consider is that of a particle whose potential energy function has the shape of a 'well' with vertical sides. It is depicted in Fig. 2.2 and is defined by

$$
\begin{aligned}
V(x) &= 0 && \text{for} \quad x < -a && \text{(Region I)} \\
V(x) &= -V_0 && \text{for} \quad -a < x < a && \text{(Region II)} \qquad (2.52) \\
V(x) &= 0 && \text{for} \quad x > a && \text{(Region III)}
\end{aligned}
$$

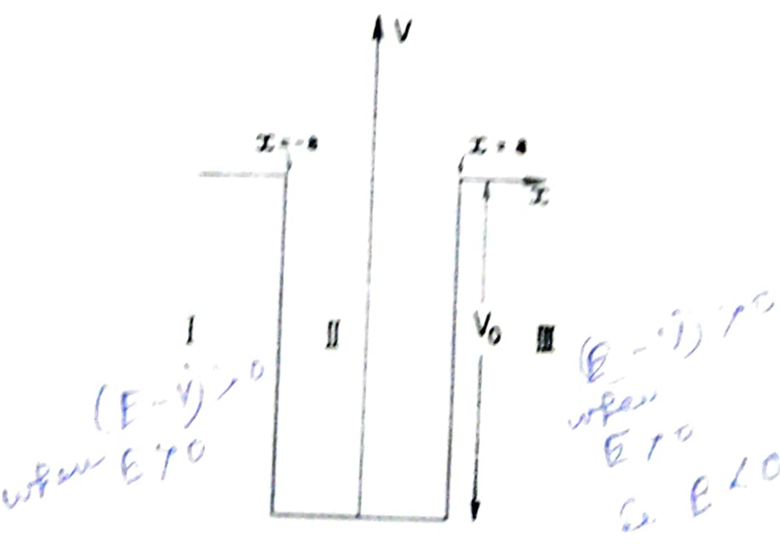

Fig. 2.2 The square well potential

If we were considering this problem in classical mechanics, we would have to keep in mind that the kinetic energy $(E - V)$ can never be negative. Since $V = 0$ for $|x| > a$, $(E - V)$ can be positive in this region only if $E > 0$. Therefore any particle with $E < 0$ cannot enter regions I and III, and will have to stay within the potential 'well', between $x = a$ and $x = -a$. We say in this case that the particle is *bound* by the potential. On the other hand if the particle has energy $E > 0$, it can go anywhere; it merely experiences momentary forces on crossing the points $x = -a$ and $x = +a$.

Let us now consider the quantum mechanical picture of this system. We confine our attention, for the present, to stationary states, which are described by solutions of Eq. 2.49. In the present case this equation, reduced to one dimension, takes distinct forms in the different regions:

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dx^2} = Eu, \qquad |x| > a \tag{2.53a}$$

and

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dx^2} - V_0 u = Eu, \; |x| < a \tag{2.53b}$$

The set of Eqs. 2.53 can be trivially solved. The nature of the solutions depends on whether $E < 0$ or $E > 0$. The former condition corresponds to bound states, as we shall see, and we take up this case first.

2.11 Bound States in a Square Well: ($E < 0$)

(a) *Admissible Solutions of Wave Equation:* For $E < 0$, we write

$$\frac{2mE}{\hbar^2} = -\alpha^2 \text{ and } \frac{2m(E+V_0)}{\hbar^2} = \beta^2;\ \alpha, \beta > 0 \tag{2.54}$$

Eqs. 2.53 can then be conveniently rewritten in terms of the positive constants α^2 and β^2 as

$$\frac{d^2u}{dx^2} - \alpha^2 u = 0, \quad |x| > a \tag{2.55a}$$

$$\frac{d^2u}{dx^2} + \beta^2 u = 0, \quad |x| < a \tag{2.55b}$$

The general solution of the second of these equations is obviously

$$u^{II}(x) = A\cos\beta x + B\sin\beta x \tag{2.56}$$

where A and B are any constants. The superscript II on $u(x)$ here indicates that it is the solution valid in Region II i.e. ($|x| < a$). The equation which holds in Region I, ($-\infty < x < a$), is (2.55*a*). Its general solution is a linear combination of its two independent solutions, $e^{\alpha x}$ and $e^{-\alpha x}$. However the latter *does not remain finite* everywhere in Region I. In fact, as $x \to -\infty$, it becomes infinitely large. Therefore the only admissible solution in Region I must be of the form

$$u^{I}(x) = C\,e^{\alpha x} \tag{2.57}$$

Region III, ($a < x < \infty$), is also governed by Eq. 2.55*a* with the idependent solutions $e^{\alpha x}$ and $e^{-\alpha x}$. But now it is $e^{-\alpha x}$ which remains well behaved everywhere and $e^{\alpha x}$ which is not admissible (since it diverges as $x \to +\infty$). Therefore, we have

$$u^{III}(x) = D\,e^{-\alpha x} \tag{2.58}$$

where D, like A, B, and C, is an undetermined constant.

Thus the solution $u(x)$ has three different forms u^{I}, u^{II}, u^{III} in the three regions. We must now make sure that $u(x)$ and its first derivative (du/dx) are continuous everywhere,[8] as demanded by the condition (c) of Sec. 2.8. In particular, at the point $x = -a$ where Regions I and II meet, we should have

$$u^{I} = u^{II} \text{ and } \frac{du^{I}}{dx} = \frac{du^{II}}{dx}, \quad (x = -a) \tag{2.59}$$

or explicitly,

$$Ce^{-\alpha a} = A\cos\beta a - B\sin\beta a, \quad C\alpha e^{-\alpha a} = A\beta\sin\beta a + B\beta\cos\beta a \tag{2.60}$$

[8] In view of the relation (2.50) between $\psi(x, t)$ and $u(x)$, it is obvious that all the admissibility conditions on ψ must be equally satisfied by $u(x)$.

Similarly, at $x = a$, where Regions II and III meet, we must have

$$u^{II} = u^{III} \text{ and } \frac{du^{II}}{dx} = \frac{du^{III}}{dx}, \quad (x = a) \tag{2.61}$$

These lead to

$$De^{-\alpha a} = A\cos\beta a + B\sin\beta a, \quad -D\alpha e^{-\alpha a} = -A\beta\sin\beta a + B\beta\cos\beta a \tag{2.62}$$

From Eqs. 2.60 and 2.62 we readily find that

$$2A\cos\beta a = (C + D)\,e^{-\alpha a} \tag{2.63a}$$
$$2A\beta\sin\beta a = (C + D)\,\alpha e^{-\alpha a} \tag{2.63b}$$
$$2B\sin\beta a = -(C - D)\,e^{-\alpha a} \tag{2.64a}$$
$$2B\beta\cos\beta a = (C - D)\,\alpha e^{-\alpha a} \tag{2.64b}$$

Eqs. 2.63 show that if $(C + D) \neq 0$, then $A \neq 0$, and further,

$$\alpha = \beta\tan\beta a \tag{2.65a}$$

This relation implies that

$$C = D \text{ and } B = 0 \tag{2.65b}$$

and hence using the relation in eqn (2.63a)

$$D = Ae^{\alpha a}\cos\beta a \tag{2.65c}$$

To verify this, we substitute for the factor α in Eq. 2.64b the value determined above; then, multiplying the equation by $\sin\beta a$ and using Eq. 2.64a, we obtain $-(C - D)\cos^2\beta a = (C - D)\sin^2\beta a$. Since $\sin^2\beta a$ cannot be equal to the negative quantity $-\cos^2\beta a$, we must have $C = D$. Thus we obtain Eq. 2.65b, and on feeding this back into Eq. 2.63, Eq. 2.65c follows.

Eqs. 2.65 give one type of solution for our problem. Another type of solution exists for $C \neq D$ and $B \neq 0$, when we get from Eqs. 2.64

$$\alpha = -\beta\cot\beta a. \tag{2.66a}$$

Repetition of the kind of arguments used above will show that now

$$C = -D \text{ and } A = 0 \tag{2.66b}$$

and

$$D = Be^{\alpha a}\sin\beta a \tag{2.66c}$$

(*b*) *The Energy Eigenvalues—Discrete Spectrum:* We can now see that *both the types of solutions exist only for certain discrete values of the energy parameter E.* We observe first of all that by virtue of Eq. 2.54

$$(\alpha^2 + \beta^2)\,a^2 = \frac{2mV_0a^2}{\hbar^2} = \frac{V_0}{\Delta} \tag{2.67}$$

where

$$\Delta = \frac{\hbar^2}{2ma^2} = \frac{(\hbar/a)^2}{2m} \tag{2.68}$$

The parameter Δ has an interesting interpretation. The half-width a of the potential well indicates the uncertainty in the position of a particle confined to the well, and associated with this, there is an uncertainty of the order of $(\hbar/a)$ in the momentum. The corresponding energy $\Delta = (\hbar/a)^2/2m$ is a natural unit in terms of which the depth of the potential may be measured. Thus the non-dimensional parameter (V_0/Δ) of Eq. 2.67 is a measure of the *strength* of the potential.

Now, in the case of solutions of the first type, we have, besides Eq. 2.67, the

further relation (2.65a) between α and β, which has two consequences. One is that since α and β have been defined to be positive, $(\alpha/\beta) = \tan \beta a$ must be positive and hence only values of βa lying in the intervals

$$2r \frac{\pi}{2} \leqslant \beta a \leqslant (2r+1) \frac{\pi}{2} \quad (r = 0,1,2,\ldots) \tag{2.69a}$$

are admissible. Secondly, the substitution of $\alpha = \beta \tan \beta a$ into Eq. 2.67 leads to the requirement

$$\frac{V_0}{\Delta} = \beta^2 a^2 \sec^2 \beta a, \text{ or } \left(\frac{\Delta}{V_0}\right)^{1/2} \beta a = |\cos \beta a| \tag{2.69b}$$

The modulus sign arises because the left hand side of the equation is known to be positive.

Similarly, for the second type of solutions of the wave equation, given by (2.66), we find from Eqs. 2.66a and 2.67 that

$$(2r-1) \frac{\pi}{2} \leqslant \beta a \leqslant 2r \frac{\pi}{2}, \quad (r = 1,2,\ldots) \tag{2.70a}$$

and

$$\frac{V_0}{\Delta} = \beta^2 a^2 \operatorname{cosec}^2 \beta a, \text{ or } \left(\frac{\Delta}{V_0}\right)^{1/2} \beta a = |\sin \beta a| \tag{2.70b}$$

Eqs. 2.69 and 2.70 can be satisfied only by certain specific discrete values of β, which can be found graphically. These special values β_n are determined by the intersections of the straight line $(\Delta/V_0)^{1/2}\beta a$ with the curves for $|\cos \beta a|$ and $|\sin \beta a|$. The parts of $|\cos \beta a|$ and $|\sin \beta a|$ which lie within the respective allowed intervals—conditions (2.69a) and (2.70a)—are shown as solid lines and dashed lines respectively in Fig. 2.3. The parts to be ignored are indicated by dotted lines.

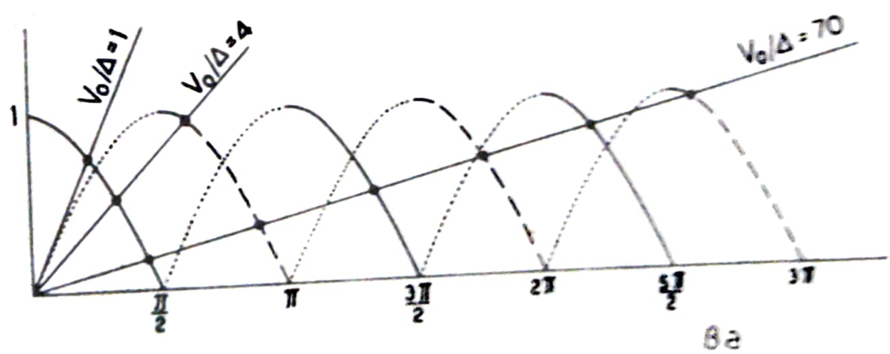

Fig. 2.3 Graphical solution for allowed values of β

If the intersections occur at $\beta = \beta_n$ ($n = 0, 1, 2, \ldots$) the corresponding allowed values of the energy are obtained from Eq. (2.54) as

$$E_n = \frac{\hbar^2}{2m}\beta_n^2 - V_0 = \left[(\beta_n a)^2 \frac{\Delta}{V_0} - 1\right] V_0 \tag{2.71}$$

It may be noted that the value of the combination of parameters appearing in the square bracket may be read off directly from the figure. It is evident that the number of these energy levels is finite. Inspection of Fig. 2.3 shows

that if $(\Delta/V_0)^{1/2}\,\beta a$ reaches the value unity for a value of βa in the interval $\frac{1}{2}\pi N \leqslant \beta a < \frac{1}{2}\pi(N+1)$, then there are $(N+1)$ intersections. In other words, the number of discrete energy levels is $(N+1)$ if $\frac{1}{2}\pi N(\Delta/V_0)^{1/2} \leqslant 1 < \frac{1}{2}\pi(N+1)(\Delta/V_0)^{1/2}$, that is, if

$$N \leqslant \frac{2}{\pi}\left(\frac{V_0}{\Delta}\right)^{1/2} < (N+1) \tag{2.72}$$

It is noteworthy that there exists at least one bound state, however weak the potential may be.

(c) *The Energy Eigenfunctions; Parity*: We observe that the energy levels E_n with $n = 0, 2, 4, \ldots$ correspond to solutions characterized by Eqs. 2.65. In this case any solution $u_n(x)$ has the following explicit forms in the three regions:

$$\left.\begin{aligned} u_n^{\mathrm{I}}(x) &= (A\,e^{\alpha_n a}\cos\beta_n a)\,e^{\alpha_n x} && (x < -a)\\ u_n^{\mathrm{II}}(x) &= A\cos\beta_n x && (-a < x < a)\\ u_n^{\mathrm{III}}(x) &= (A\,e^{\alpha_n a}\cos\beta_n a)\,e^{-\alpha_n x} && (x > a)\\ & && (n = 0, 2, \ldots)\end{aligned}\right\} \tag{2.73}$$

The nature of such functions is illustrated graphically in Fig. 2.4(a). It is evident that $u_n(x)$ is symmetric about the origin:

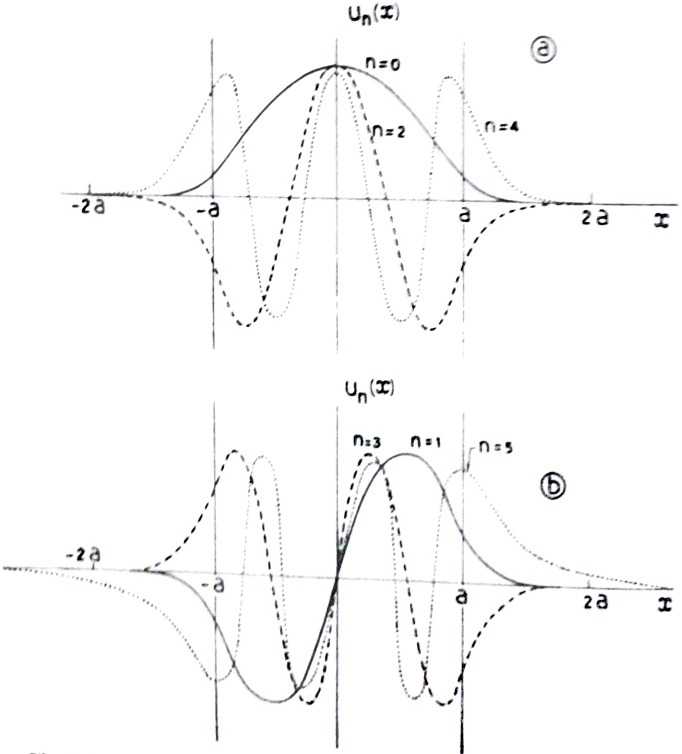

Fig. 2.4 (a) Even parity eigenfunctions; and (b) Odd parity eigenfunctions

$$u_n(x) = u_n(-x) \tag{2.74}$$

In general, any wave function which has this symmetry property is said to be of *even parity*.

The eigenfunctions corresponding to $n = 1,3,\ldots$ are characterized by Eqs. 2.66. For these we have, explicitly,

$$\left.\begin{aligned} u_n^{\mathrm{I}}(x) &= -(B e^{\alpha_n a} \sin\beta_n a)\, e^{\alpha_n x} && (x < -a) \\ u_n^{\mathrm{II}}(x) &= B \sin\beta_n x && (-a < x < a) \\ u_n^{\mathrm{III}}(x) &= (B e^{\alpha_n a} \sin\beta_n a)\, e^{-\alpha_n x} && (x > a) \\ & && (n = 1,3,\ldots). \end{aligned}\right\} \tag{2.75}$$

These functions are illustrated in Fig. 2.4(b). They are antisymmetric with respect to the origin, i.e.,

$$u_n(x) = -u_n(-x) \tag{2.76}$$

Any wave function which has this property of antisymmetry is said to be of *odd parity*.

Thus the eigenfunctions describing the stationary states of a particle in a square well potential, when considered in order of increasing energy, are alternately of even and odd parity. The fact that the eigenfunctions have even or odd parity is a consequence of the symmetry of the potential V itself with respect to the origin. The proof of this statement, as well as the definition of parity in the case of more general systems (many particles, in three dimensions) will be given in Sec. 4.11.

(d) *Penetration Into Classically Forbidden Regions:* The wave functions (2.74) and (2.75) provide an illustration of a feature of quantum mechanics which is of fundamental importance. We recall that as discussed at the beginning of this section, a classical particle of energy $E < 0$ can stay only in Region II and cannot at all enter Regions I and III. However, the quantum mechanical wave functions $u(x)$ have nonvanishing values in both these classically forbidden regions. Therefore, according to the probability interpretation, there exists a nonvanishing probability that the particle is somewhere within these regions. However as one goes from the boundary point ($x = -a$ or $x = +a$) deeper into the forbidden region, the probability density $|u(x)|^2$ decreases rapidly, (proportional to $e^{-2\alpha|x|}$) to zero. Therefore, the particle cannot escape to infinitely large distances; it stays bound to the potential well. Thus all the states which we have so far considered ($E < 0$) are *bound states*.

Example 2.7.—The eigenfunctions (2.73) are normalized if we take $A = (a + \alpha^{-1})^{-1/2}$. Verify this. The probability of finding the particle in the classically forbidden regions is

$$\int_{-\infty}^{-a} (u^{\mathrm{I}})^2\, dx + \int_a^{\infty} (u^{\mathrm{III}})^2\, dx = 2\int_a^{\infty} (u^{\mathrm{III}})^2\, dx = (\cos^2\beta a)/(1 + \alpha a)$$

Example 2.8.—Eq. 2.71 gives the positions of the energy levels, as measured from the bottom of the potential well, as $E_n + V_0 = (\beta_n a)^2\Delta$. For a very deep potential well ($V_0 \to \infty$) we see from Fig. 2.3 that $\beta a \to (n+1)\pi/2$. Hence the energy levels in this case are given by $E_n + V_0 = \frac{1}{4}\pi^2\Delta(n+1)^2$.

Further, in this limit the wave function in the classically forbidden regions tends to zero, as can be seen from Eqs. 2.73 and 2.75.

2.12 The Square Well: Non-localized States ($E > 0$)

When $E > 0$, $2mE/\hbar^2$ is positive and, therefore, instead of Eq. 2.54 we write

$$\frac{2mE}{\hbar^2} = k^2 \text{ and } \frac{2m(E - V_0)}{\hbar^2} = \beta^2, \quad (k, \beta > 0) \tag{2.77}$$

Clearly, the only change in the Schrödinger equations 2.55 is that $-\alpha^2$ is to be replaced by $+k^2$. The possible independent solutions in Regions I and III now become e^{ikx} and e^{-ikx} instead of $e^{\alpha x}$ and $e^{-\alpha x}$. But unlike the latter pair, of which one becomes infinite as $x \to +\infty$ or $x \to -\infty$, both e^{ikx} and e^{-ikx} remain finite. Therefore, both have to be retained in the general solution, and instead of Eqs. 2.57 and 2.58 we now have

$$u^{I}(x) = C_+ e^{ikx} + C_- e^{-ikx} \tag{2.78a}$$

$$u^{III}(x) = D_+ e^{ikx} + D_- e^{-ikx} \tag{2.78b}$$

where C_+, C_-, D_+, D_- are undetermined constants. In Region II the same Eq. 2.55a holds as before, but we will find it convenient to rewrite its general solution (2.56) in a form similar to u^I and u^{III}:

$$u^{II}(x) = A_+ e^{i\beta x} + A_- e^{-i\beta x} \tag{2.78c}$$

where $A_+ = \frac{1}{2}(A - iB)$ and $A_- = \frac{1}{2}(A + iB)$. A particle with the wave function (2.78) is evidently *not localized*: it is not confined to any finite region of space, since $|u(x)|^2$ remains nonzero even when $x \to \pm\infty$. Such wave functions are of course not normalizable.

The forms u^I, u^{II}, u^{III} which define $u(x)$ in the different regions must be subjected to the continuity conditions (2.59) and (2.61) at $x = -a$ and $x = +a$ respectively. However now, unlike in the case of bound states, this procedure *does not lead to any restrictions on k or β. Hence any energy $E > 0$ is an eigenvalue.* The reason for the dramatic difference between the cases $E > 0$ and $E < 0$ can be seen very simply. For $E < 0$ we had *four* constants A, B, C, D and an equal number of *linear homogeneous* equations for them, arising from the continuity conditions. Under such circumstances it is well known that nontrivial solutions exist only if the determinant formed by the coefficients appearing in the equations is zero. Eqs. 2.65*a* and 2.66*a* merely express this restriction on the coefficients, which in turn forces E to take only a discrete set of values. On the other hand, when $E > 0$, the continuity conditions still give four linear homogeneous equations but now they involve *six* unknowns $A_\pm, C_\pm, D_\pm$. Since the number of equations is less than the number of unknowns, an infinite number of solutions exist, whatever the coefficients in the equations may be.

As the coefficients depend on E, this amounts to saying that admissible solutions exist for *every* $E > 0$. Thus the energy eigenvalues form a *continuous* (not a discrete) set. We say that the energy spectrum (for $E > 0$) is a *continuum.*

To complete the determination of the eigenfunctions belonging to any $E > 0$, we have to obtain explicitly the relations among the coefficients